

$$23\% \text{ of } 52 = ???$$

ACCUPLACER MATH REVIEW LESSONS

$$4x - 10 = 2$$
$$x = ???$$

By: Joe Cleary

Edited in 2009 by:
Jack Keating and
Marianne Rosato



Table of Contents

*	<u>Fractions</u>	Pgs 3-14
*	<u>Decimals</u>	Pgs 15-23
*	<u>Percentages</u>	Pgs 24-28
*	<u>Ratio and Proportion</u>	Pgs 29-32
*	<u>Signed Numbers</u>	Pgs 33-39
*	<u>Exponents</u>	Pgs 40-44
*	<u>Algebraic Expressions</u>	Pgs 45-52
*	<u>Factoring</u>	Pgs 53-57
*	<u>Equations and Inequalities</u>	Pgs 58-68

Fractions

FRACTIONS

A fraction is the quotient of two quantities and is of the form, $\frac{a}{b}$, where a and b are any numbers, $b \neq 0$. a is the numerator and b is the denominator

Fractions are either proper or improper:

Proper Fraction: the numerator is less than the denominator

Examples: $\frac{3}{5}$, $\frac{7}{10}$

Improper Fraction: the numerator is greater than or equal to the denominator

Examples: $\frac{5}{3}$, $\frac{9}{5}$, $\frac{4}{4}$

Equivalent Fractions: $\frac{a}{b}$ is equivalent to $\frac{c}{d}$ or $\frac{a}{b} = \frac{c}{d}$ if $ad = bc$

Example:

$$\frac{3}{4} = \frac{6}{8} \text{ because}$$
$$3 \times 8 = 4 \times 6$$
$$24 = 24$$

Equivalent fractions are formed by:

(1) Multiplying both the numerator and denominator by the same nonzero number, i.e.

$$\frac{a}{b} = \frac{an}{bn}$$

Examples:

(a) $\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$

(b) $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$

(2) Dividing both the numerator and denominator by the same nonzero number, i.e.

$$\frac{a}{b} = \frac{a \div n}{b \div n}$$

Examples:

(a) $\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$

(b) $\frac{12}{14} = \frac{12 \div 2}{14 \div 2} = \frac{6}{7}$

Problems

Find the missing value of x.

$$(1) \frac{2}{3} = \frac{x}{18}$$

$$(7) \frac{12}{15} = \frac{x}{5}$$

$$(2) \frac{4}{5} = \frac{x}{20}$$

$$(8) \frac{20}{30} = \frac{x}{3}$$

$$(3) \frac{1}{4} = \frac{x}{12}$$

$$(9) \frac{7}{21} = \frac{x}{3}$$

$$(4) \frac{3}{8} = \frac{6}{x}$$

$$(10) \frac{10}{16} = \frac{5}{x}$$

$$(5) \frac{5}{9} = \frac{15}{x}$$

$$(11) \frac{14}{35} = \frac{2}{x}$$

$$(6) \frac{2}{7} = \frac{10}{x}$$

$$(12) \frac{18}{24} = \frac{3}{x}$$

Answers

- (1) 12 (2) 16 (3) 3 (4) 16 (5) 27 (6) 35 (7) 4 (8) 2 (9) 1 (10) 8
(11) 5 (12) 4

Mixed Numbers

A mixed number is a whole number added to a proper fraction

Examples:

$$(1) 1 + \frac{1}{2} = 1\frac{1}{2}$$

$$(2) 2 + \frac{3}{5} = 2\frac{3}{5}$$

$$(3) 3 + \frac{1}{4} = 3\frac{1}{4}$$

We can convert mixed numbers to improper fractions and improper fractions to mixed numbers.

To convert a mixed number to an improper fraction:

- (1) Multiply the whole number by the denominator of the fraction
- (2) Add the numerator of the fraction to the product
- (3) Place your answer over the denominator of the fraction

Examples:

(a) Convert $2\frac{3}{5}$ to an improper fraction

(1) $2 \times 5 = 10$

(2) $10 + 3 = 13$

(3) $\frac{13}{5}$

(b) Convert $3\frac{1}{4}$ to an improper fraction

(1) $3 \times 4 = 12$

(2) $12 + 1 = 13$

(3) $\frac{13}{4}$

To convert an improper fraction to a mixed number

(1) Divide the numerator by the denominator. This gives the whole number part of the answer

(2) The remainder, if any, is placed over the denominator. This forms the fractional part of the answer.

Examples:

(a) Convert $\frac{9}{4}$ to a mixed number

(1)

$$\begin{array}{r} 2 \\ 4 \overline{)9} \\ \underline{8} \\ 1 \end{array}$$

(2) $2\frac{1}{4}$

(b) Convert $\frac{10}{5}$ to a mixed number

(1)

$$\begin{array}{r} 2 \\ 5 \overline{)10} \\ \underline{10} \\ 0 \end{array}$$

(2) 2

Prime Number: an integer greater than 1 that is divisible only by itself and 1.

Examples: 2 (the only even prime), 3, 5, 7, 11, 13, 17, 19, 23...

Composite Number: an integer greater than 1 that is divisible by a number other than itself and 1

Examples: 4, 6, 8, 9, 10, 12, 14, 15...

4 is not only divisible by itself and one it is also divisible by 2.

Any composite number can be written as a product of prime numbers or prime factors (numbers that are multiplied together):

Examples:

(a)

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$2 \overline{)12}$$

$$2 \overline{)6}$$

$$3 \overline{)3}$$

$$1$$

(b)

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$$

$$2 \overline{)24}$$

$$2 \overline{)12}$$

$$2 \overline{)6}$$

$$3 \overline{)3}$$

$$1$$

(c)

$$75 = 3 \times 5 \times 5 = 3 \times 5^2$$

$$3 \overline{)75}$$

$$5 \overline{)25}$$

$$5 \overline{)5}$$

$$1$$

To Find the Least Common Denominator (LCD)

(1) Write each denominator as a product of its prime numbers

(2) Multiply the highest power of each prime number

Example:

(a) Find the LCD of 6 and 9

(1) $6 = 2 \times 3$ $9 = 3 \times 3 = 3^2$

(2) $\text{LCD} = 2 \times 3^2 = 2 \times 9 = 18$

(b) Find the LCD of 8 and 12

(1) $8 = 2 \times 4 = 2 \times 2 \times 2 = 2^3$

$12 = 2 \times 6 = 2 \times 2 \times 3 = 2^2 \times 3$

(2) $\text{LCD} = 2^3 \times 3 = 8 \times 3 = 24$

(c) Find the LCD of 10, 15, and 20

(1) $10 = 2 \times 5$ $15 = 3 \times 5$ $20 = 2 \times 10 = 2 \times 2 \times 5 = 2^2 \times 5$

(2) $\text{LCD} = 2^2 \times 3 \times 5 = 60$

Problems

Find the LCD of

(1) 12 and 18

(2) 9 and 15

(3) 6, 9, and 18

Answers

(1) 36 (2) 45 (3) 18

Addition and Subtraction of Fractions

(1) $\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$

If the denominators of the two fractions that are to be combined by addition or subtraction are the same, combine the numerators and place the answer over the common denominator. Reduce the answer to simplest terms if possible.

Examples:

(a) $\frac{4}{9} + \frac{2}{9} = \frac{4+2}{9} = \frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$

(b) $\frac{7}{12} - \frac{5}{12} = \frac{7-5}{12} = \frac{2}{12} = \frac{2 \div 2}{12 \div 2} = \frac{1}{6}$

(c) $\frac{9}{14} + \frac{3}{14} - \frac{5}{14} = \frac{9+3-5}{14} = \frac{7}{14} = \frac{7 \div 7}{14 \div 7} = \frac{1}{2}$

(d) $\frac{7}{15} - \frac{4}{15} + \frac{8}{15} = \frac{7-4+8}{15} = \frac{11}{15}$

(2) $\frac{a}{b} \pm \frac{c}{d} = \frac{ad}{bd} \pm \frac{bc}{bd} = \frac{ad \pm bc}{bd}$

If the denominators of the fractions that are to be combined by addition or subtraction differ:

(A) Find a LCD

- (B) Rewrite each fraction as an equivalent fraction with the LCD
 (C) Combine the numerators and place the answer over the LCD
 (D) Reduce the answer to simplest terms

Examples:

(a)

$$\frac{1}{6} + \frac{5}{8} \quad \text{LCD} = 24$$

$$\frac{4}{24} + \frac{15}{24} = \frac{4+15}{24} = \frac{19}{24}$$

(b)

$$\frac{5}{6} - \frac{4}{9} \quad \text{LCD} = 18$$

$$\frac{15}{18} - \frac{8}{18} = \frac{15-8}{18} = \frac{7}{18}$$

(c)

$$\frac{1}{5} + \frac{1}{6} - \frac{3}{10} \quad \text{LCD} = 30$$

$$\frac{6}{30} + \frac{5}{30} - \frac{9}{30} = \frac{6+5-9}{30} = \frac{2}{30} = \frac{1}{15}$$

(d)

$$\frac{5}{12} - \frac{1}{4} + \frac{4}{9}$$

$$\frac{15}{36} - \frac{9}{36} + \frac{16}{36} = \frac{15-9+16}{36} = \frac{22}{36} = \frac{11}{18}$$

(e)

$$3\frac{1}{4} + 2\frac{1}{6} = \quad \text{LCD} = 12$$

(add the fractional parts then the whole number parts)

$$3\frac{3}{12} + 2\frac{2}{12} = 5\frac{5}{12}$$

(f)

$$9\frac{5}{8} - 3\frac{5}{12}$$

$$9\frac{15}{24} - 3\frac{10}{24} = 6\frac{5}{24}$$

(g)

$$4\frac{1}{3} - 2\frac{2}{5} \quad LCD = 15$$

$$4\frac{5}{15} - 2\frac{6}{15} = 3\frac{20}{15} - 2\frac{6}{15} = 1\frac{14}{15}$$

Since $\frac{6}{15}$ is greater than $\frac{5}{15}$ you cannot subtract

“larger” from “smaller” so

borrow 1 from the 4, change the 1 to $\frac{15}{15}$,

and add to $\frac{5}{15}$

$$\frac{15}{15} + \frac{5}{15} = \frac{20}{15}$$

Problems

Perform the indicated operations and **leave your answers in simplest form.**

(1) $\frac{5}{12} + \frac{1}{12}$

(8) $\frac{7}{15} - \frac{3}{10}$

(2) $\frac{7}{15} + \frac{4}{15} + \frac{1}{15}$

(9) $\frac{7}{15} - \frac{7}{20}$

(3) $\frac{11}{12} - \frac{1}{12}$

(10) $5\frac{1}{2} + 3\frac{1}{6}$

(4) $\frac{13}{20} - \frac{7}{20}$

(11) $9\frac{1}{3} + 2\frac{1}{4} + 3\frac{1}{5}$

(5) $\frac{3}{4} + \frac{1}{5}$

(12) $8\frac{5}{6} - 3\frac{1}{4}$

(6) $\frac{1}{6} + \frac{3}{10} + \frac{1}{15}$

(13) $7\frac{3}{10} - 4\frac{7}{15}$

(7) $\frac{1}{12} + \frac{11}{18}$

(14) A student completed $\frac{1}{3}$ of a paper one day and $\frac{1}{2}$ the next day. How much of the paper did he complete and how much is left for him to complete?

(15) A marathoner strives to run 30 miles per week. She runs $5\frac{1}{2}$ miles, $3\frac{1}{4}$ miles, $2\frac{5}{6}$ miles and $6\frac{3}{8}$ miles. How many miles has she run and how many miles must she run to reach her goal?

Answers

(1) $\frac{1}{2}$

(8) $\frac{1}{6}$

(2) $\frac{4}{5}$

(9) $\frac{7}{60}$

(3) $\frac{5}{6}$

(10) $8\frac{2}{3}$

(4) $\frac{3}{10}$

(11) $14\frac{47}{60}$

(5) $\frac{19}{20}$

(12) $5\frac{7}{12}$

(6) $\frac{8}{15}$

(13) $2\frac{5}{6}$

(7) $\frac{25}{36}$

(14) He completed $\frac{5}{6}$ of the paper.

He has $\frac{1}{6}$ of the paper left to complete.

(15) She has run $17\frac{23}{24}$ miles.

She has $12\frac{1}{24}$ miles left to run.

Multiplying Fractions

Let $\frac{a}{b}$ and $\frac{c}{d}$ represent two fractions: $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

To Multiply Fractions:

- (A) Multiply the numerators
- (B) Multiply the denominators
- (C) Reduce the product to simplest terms

Examples:

$$(a) \frac{8}{9} \times \frac{2}{3} = \frac{8 \times 2}{9 \times 3} = \frac{16}{27}$$

$$(b) \frac{5}{6} \times \frac{1}{2} = \frac{5 \times 1}{6 \times 2} = \frac{5}{12}$$

$$(c) \frac{4}{9} \times \frac{3}{10} = \frac{4 \times 3}{9 \times 10} = \frac{12}{90} = \frac{2}{15}$$

In Example (c) we can simplify the fraction first and then multiply.

$$\frac{4}{9} \times \frac{3}{10} = \frac{2}{3} \times \frac{1}{5} = \frac{2 \times 1}{3 \times 5} = \frac{2}{15}$$

$$(d) \frac{9}{10} \times \frac{5}{12} = \frac{3}{2} \times \frac{1}{4} = \frac{3 \times 1}{2 \times 4} = \frac{3}{8}$$

$$(e) 3\frac{1}{2} \times \frac{4}{5} = \frac{7}{2} \times \frac{4}{5} = \frac{7}{1} \times \frac{2}{5} = \frac{7 \times 2}{1 \times 5} = \frac{14}{5} = 2\frac{4}{5}$$

Convert mixed numbers to improper fractions, then multiply.

$$(f) 2\frac{4}{5} \times 3\frac{1}{3} = \frac{14}{5} \times \frac{10}{3} = \frac{14}{1} \times \frac{2}{3} = \frac{14 \times 2}{1 \times 3} = \frac{28}{3} = 9\frac{1}{3}$$

$$(g) 2\frac{2}{3} \times 3\frac{1}{2} \times 1\frac{1}{5} = \frac{8}{3} \times \frac{7}{2} \times \frac{6}{5} = \frac{4}{1} \times \frac{7}{1} \times \frac{2}{5} = \frac{4 \times 7 \times 2}{1 \times 1 \times 5} = \frac{56}{5} = 11\frac{1}{5}$$

Dividing Fractions

Let $\frac{a}{b}$ and $\frac{c}{d}$ represent any two fractions: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

- (A) Invert the divisor (the fraction immediately following the division sign)
- (B) Multiply the fractions
- (C) Reduce the answer to simplest terms

Examples:

$$(a) \frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{2 \times 4}{3 \times 3} = \frac{8}{9}$$

$$(b) \frac{8}{9} \div \frac{4}{3} = \frac{8}{9} \times \frac{3}{4} = \frac{2}{3} \times \frac{1}{1} = \frac{2 \times 1}{3 \times 1} = \frac{2}{3}$$

$$(c) \frac{7}{12} \div \frac{21}{4} = \frac{7}{12} \times \frac{4}{21} = \frac{1}{3} \times \frac{1}{3} = \frac{1 \times 1}{3 \times 3} = \frac{1}{9}$$

$$(d) \frac{3}{8} \div \frac{15}{4} = \frac{3}{8} \times \frac{4}{15} = \frac{1}{2} \times \frac{1}{5} = \frac{1 \times 1}{2 \times 5} = \frac{1}{10}$$

$$(e) 6\frac{1}{4} \div \frac{5}{8} = \frac{25}{4} \div \frac{5}{8} = \frac{25}{4} \times \frac{8}{5} = \frac{5}{1} \times \frac{2}{1} = \frac{5 \times 2}{1 \times 1} = \frac{10}{1} = 10$$

Change mixed numbers to improper fractions, then divide.

$$(f) 5\frac{1}{4} \div 3\frac{1}{2} = \frac{21}{4} \div \frac{7}{2} = \frac{21}{4} \times \frac{2}{7} = \frac{3}{2} \times \frac{1}{1} = \frac{3 \times 1}{2 \times 1} = \frac{3}{2} = 1\frac{1}{2}$$

$$(g) \frac{\frac{6}{7}}{\frac{8}{21}} = \frac{6}{7} \div \frac{8}{21} = \frac{6}{7} \times \frac{21}{8} = \frac{9}{4} = 2\frac{1}{4}$$

Problems

Perform the indicated operations and leave your answers in simplest form.

$$(1) \frac{7}{12} \times \frac{6}{5}$$

$$(7) \frac{5}{9} \div \frac{25}{18}$$

$$(2) \frac{8}{9} \times \frac{5}{6}$$

$$(8) 3 \div \frac{3}{2}$$

$$(3) \frac{4}{7} \times 2 \times \frac{7}{8}$$

$$(9) \frac{5}{9} \div 10$$

$$(4) 4\frac{2}{3} \times 1\frac{5}{7}$$

$$(10) 3\frac{3}{4} \div 5\frac{1}{2}$$

$$(5) 2\frac{1}{5} \times 3\frac{1}{3} \times \frac{9}{22}$$

$$(11) 4\frac{2}{5} \div 2\frac{3}{4}$$

$$(6) \frac{7}{8} \div \frac{5}{4}$$

$$(12) 5\frac{5}{6} \div 5$$

- (13) A graduating class consists of 90 seniors of which $\frac{2}{5}$ are business majors. How many business majors are graduating?
- (14) A person earns \$1000 per week. He pays $\frac{1}{5}$ in taxes, $\frac{1}{20}$ in health insurance, $\frac{1}{10}$ in tax-deferred savings. How much of his salary does he take home?
- (15) A chef has $4\frac{1}{2}$ teaspoons of vanilla. Each batch of cookies require $1\frac{1}{4}$ teaspoons. How many batches of cookies can he make?
- (16) A car averages $25\frac{1}{2}$ miles per gallon of gas on a 561 mile trip. How many gallons of gas does the car use for the trip.

Answers

- | | |
|---------------------|---|
| (1) $\frac{7}{10}$ | (9) $\frac{1}{18}$ |
| (2) $\frac{20}{27}$ | (10) $\frac{15}{22}$ |
| (3) 1 | (11) $\frac{8}{5} = 1\frac{3}{5}$ |
| (4) 8 | (12) $\frac{7}{6} = 1\frac{1}{6}$ |
| (5) 3 | (13) 36 business majors are graduating. |
| (6) $\frac{7}{10}$ | (14) \$650 |
| (7) $\frac{2}{5}$ | (15) 3 batches (with $\frac{3}{5}$ tsp vanilla left over) |
| (8) 2 | (16) The car uses 22 gal of gas for the trip. |

Decimals

DECIMALS

A number in decimal form has:

- (1) A whole number portion
- (2) A fractional portion

These two portions are separated by the decimal point: 8.75 (8 is the whole number portion and 75 is the fractional portion)

If a number does not have a decimal point it is understood to have one to the right of the last digit: $5 = 5.$

Place Value

10,000 1,000 100 10 1 $\frac{1}{10}$ $\frac{1}{100}$ $\frac{1}{1,000}$ $\frac{1}{10,000}$

As we move from left to right, place values are getting smaller

3 **2** . **7** **6** **5**
tens ones decimal tenths hundredths thousandths

Ordering Decimals

We may want to arrange decimals from smallest to largest or largest to smallest.

To Compare Decimals

Make a place by place comparison by

- (1) Aligning the numbers according to the decimal point
- (2) Comparing the digits that have the same place value
- (3) Determine the place value with the largest digit

Examples:

- (a) Determine the larger decimal number

8.235 or 8.242

8.235

8.242

Both numbers have an 8 in the ones place and 2 in the tenths place. In the hundredths place $4 > 3$. Thus $8.242 > 8.235$

- (b) Arrange the numbers in order from smallest to largest:

0.136, 0.1372, 0.048, 1.002.

First rewrite the numbers with the decimal points lined up

0.136

0.1372

0.048

1.002

In the ones places, the fourth number has a 1, the other three numbers have a 0. 1.002 is the largest. For the three remaining numbers, the first and second have a 1 in the tenths place, the third has a 0, 0.048 is the smallest. Both remaining numbers have a 3 in the tenths place. In the hundredths place $7 > 6$. Thus $0.1372 > 0.136$.

Order: 0.048, 0.136, 0.1372, 1.002

Rounding Decimals

- (1) Find the digit in the place you are rounding (block it)
- (2) Look at the digit to the right
- (3) (A) If it is less than 5, round down, round down by dropping it and any other digits to the right.
(B) If it is 5 or greater, round up by adding 1 to the blocked digit and dropping all digits to the right.

Examples:

- (a) Round 35.683 to the nearest tenth

$$35.\overline{6}83$$

first digit to the right is 8, $8 > 5$ so add 1 to 6.

Answer: 35.7

- (b) Round 5.243 to the nearest hundredth

$$5.2\overline{4}3$$

first digit to the right is 3, $3 < 5$ so drop 3

Answer: 5.24

Converting Decimals to Fractions

- (1) Place the decimal without the decimal point in the numerator of the fraction
- (2) The denominator is 1 followed by as many zeros as there are digits after the decimal point
- (3) Reduce if possible

Examples:

- (a) Convert 0.125 to an equivalent fraction

$$0.125 = \frac{125}{1000} = \frac{1}{8}$$

- (b) Convert 3.2 to an equivalent fraction

$$3.2 = \frac{32}{10} = \frac{16}{5} = 3\frac{1}{5}$$

Converting Fractions to Decimals

- (1) Divide the denominator into the numerator
 - (A) If the remainder is 0, the decimal is terminating
 - (B) If the remainder repeats, the decimal is a repeating, non-terminating decimal

Examples:

(a) Convert $\frac{3}{4}$ to a decimal

$$\begin{array}{r} .75 \\ 4 \overline{)3.00} \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$
$$\frac{3}{4} = .75$$

(b) Convert $\frac{5}{11}$ to a decimal

$$\begin{array}{r} 0.45 \\ 11 \overline{)5.00} \\ \underline{44} \\ 60 \\ \underline{55} \\ \boxed{5} \end{array}$$

Put a line over the repeating digits

$$\frac{5}{11} = 0.4\overline{5}$$

(c) Convert $\frac{7}{15}$ to a decimal

$$\begin{array}{r} 0.46 \\ 15 \overline{)7.00} \\ \underline{60} \\ \boxed{10}0 \\ \underline{90} \\ \boxed{10} \end{array}$$

Put a line over the repeating digits

$$\frac{7}{15} = 0.4\overline{6}$$

Problems

(1) Arrange in order from smallest to largest: 0.0635, 0.0621, 0.0712

(2) Arrange in order from smallest to largest: 2.468, 2.439, 3.407, 3.612

(3) Round 12.76205 to the

- (a) nearest tenth
- (b) nearest thousandth
- (c) nearest ten

(4) Round 8.7972 to the

- a. nearest hundredth
- b. nearest thousandth
- c. nearest one

(5) Convert the following decimal numbers to equivalent fractions:

- a. 0.7 (b) 0.45 (c) 0.325 (d) 2.42

(6) Convert the following fractions to equivalent decimals

- (a) $\frac{3}{8}$ (b) $\frac{5}{12}$ (c) $\frac{4}{9}$ (d) $\frac{2}{15}$

Answers

(1) 0.0621, 0.0635, 0.0712

(2) 2.439, 2.468, 3.407, 3.612

(3) (a) 12.8 (b) 12.762 (c) 10

(4) (a) 8.80 (b) 8.797 (c) 9

(5) (a) $\frac{7}{10}$ (b) $\frac{9}{20}$ (c) $\frac{13}{40}$ (d) $\frac{121}{50} = 2\frac{21}{50}$

(6) (a) 0.375 (b) $0.4\overline{16}$ (c) $0.\overline{4}$ (d) $0.\overline{13}$

Addition and Subtraction of Decimal Numbers

To Add Decimal Numbers

- (1) Write the numbers vertically with the decimal points lined up
- (2) Add as if the numbers were whole numbers
- (3) Insert the decimal point under the others

Examples:

(a) $15.672 + 4.36 + 0.154$

15.672

4.36

0.154

20.186

(b) $24.023 + 9.47 + 10$

24.023

9.47

10.0

43.493

To Subtract Decimal Numbers

- (1) Write the numbers vertically with the decimal points lined up
- (2) Subtract, adding zeros if necessary in the minuend, as if the numbers were whole numbers
- (3) Insert the decimal point under the others

Examples:

(a) $172.345 - 38.621$

172.345

- 38.621

133.724

(b) $16.3 - 12.482$

16.300

- 12.482

3.818

Problems

Perform the indicated operations

(1) $16.82 + 3.93 + 6$

(4) $26.87 - 19.75$

(2) $25.86 + 0.349 + 2.781$

(5) $206.375 - 192.483$

(3) $7.915 + 12 + 3.8$

(6) $25.2 - 18.451$

Answers

(1) 26.75

(4) 7.12

(2) 28.99

(5) 13.892

(3) 23.715

(6) 6.749

Multiplication of Decimal Numbers

(1) Multiply the numbers as if they were whole numbers

(2) Mark off, right to left, the number of decimal places equal to the total number of decimal places in the numbers being multiplied.

Examples:

(a) 35.4×6.23

$$\begin{array}{r} 35.4 \\ \times 6.23 \\ \hline 1062 \\ 708 \\ 2124 \\ \hline 220.542 \end{array}$$

1 decimal place

2 decimal places

3 decimal places

(b) 10.5×0.285

$$\begin{array}{r} 10.5 \\ \times 0.285 \\ \hline 525 \\ 840 \\ 210 \\ \hline 2.9925 \end{array}$$

1 decimal place

3 decimal places

4 decimal places

Division of Decimal Numbers

- (1) Move the decimal point in the divisor all the way to the right
- (2) Move the decimal point in the dividend the same number of places to the right as in the divisor (and directly up into the quotient)
- (3) Divide

Examples:

(a) $31.45 \div 3.7$

divisor $\rightarrow 3.7 \overline{)31.45} \leftarrow$ *dividend*

$$\begin{array}{r} 8.5 \\ 37 \overline{)314.5} \\ \underline{296} \\ 185 \\ \underline{185} \\ 0 \end{array}$$

(b) $11.040 \div 0.046$

$0.046 \overline{)11.040}$

$$\begin{array}{r} 240 \\ 046 \overline{)11040} \\ \underline{92} \\ 184 \\ \underline{184} \\ 00 \end{array}$$

(c) $17.28 \div 48$

$48 \overline{)17.28}$

$$\begin{array}{r} 0.36 \\ 48 \overline{)17.28} \\ \underline{144} \\ 288 \\ \underline{288} \\ 0 \end{array}$$

(d) $18 \div 7.74$ Round to nearest tenth

$$\begin{array}{r} 7.74 \overline{)18} \\ \underline{2.32} \\ 774 \overline{)1800.00} \\ \underline{1548} \\ 2520 \\ \underline{2322} \\ 1980 \\ \underline{1548} \\ 332 \end{array}$$

Answer: 2.3

Problems

Perform the indicated operations

(1) 3.66×2.8

(5) $33.436 \div 5.2$

(2) 0.054×1.23

(6) $0.51264 \div 8.01$

(3) 5.23×1000

(7) $62.64 \div 72$

(4) 13.81×0.1

(8) $18.56 \div 3.8$ (Round to nearest tenth)

Answers

(1) 10.248

(5) 6.43

(2) 0.06642

(6) 0.064

(3) 5230

(7) 0.87

(4) 1.381

(8) 4.9

Percents

PERCENTS

Percent (%) means one part per hundred. 1% means one part per hundred and can be expressed as $\frac{1}{100}$ or 0.01. 35% means 35 parts per hundred and can be expressed as

$$\frac{35}{100} = \frac{7}{20} \text{ or } 0.35.$$

Changing Percents to Equivalent Fractions

To change a percent to an equivalent fraction:

- (1) Replace the percent sign with $\frac{1}{100}$ and multiply
- (2) Reduce the resulting fraction to simplest form

Examples:

$$(a) 40\% = 40 \times \frac{1}{100} = \frac{40}{100} = \frac{2}{5}$$

$$(b) 15\% = 15 \times \frac{1}{100} = \frac{15}{100} = \frac{3}{20}$$

$$(c) 7\frac{1}{2}\% = 7\frac{1}{2} \times \frac{1}{100} = \frac{15}{2} \times \frac{1}{100} = \frac{15}{200} = \frac{3}{40}$$

$$(d) 66\frac{2}{3}\% = 66\frac{2}{3} \times \frac{1}{100} = \frac{200}{3} \times \frac{1}{100} = \frac{200}{300} = \frac{2}{3}$$

Changing Percents to Equivalent Decimals

To change a percent to an equivalent decimal:

- (1) Replace the percent sign with 0.01 and multiply

Examples:

$$(a) 25\% = 25 \times 0.01 = 0.25$$

$$(b) 8\% = 8 \times 0.01 = 0.08$$

$$(c) 62.5\% = 62.5 \times 0.01 = 0.625$$

$$(d) 10\frac{1}{2}\% = 10.5\% = 10.5 \times 0.01 = 0.105$$

Observe: You can also change a percent to an equivalent decimal by (1) dropping the percent sign and (2) moving the decimal point two places to the left.

$$(e) 74\% = 74.0\% = 0.74$$

Changing Fractions to Percents

To change a fraction to an equivalent percent, multiply the fraction by 100%

Examples:

$$(a) \frac{3}{5} = \frac{3}{5} \times 100\% = \frac{3}{1} \times 20\% = 60\%$$

$$(b) \frac{7}{8} = \frac{7}{8} \times 100\% = \frac{7}{2} \times 25\% = \frac{175}{2}\% = 87\frac{1}{2}\%$$

$$(c) 2\frac{1}{2} = \frac{5}{2} \times 100\% = \frac{5}{1} \times 50\% = \frac{250}{1}\% = 250\%$$

Changing Decimals to Percents

To change a decimal to an equivalent percent: move the decimal point two places to the right and add the percent sign.

Examples:

$$(a) 0.43 = 43\%$$

$$(b) 0.95 = 95\%$$

$$(c) 0.04 = 4\%$$

$$(d) 0.105 = 10.5\%$$

Problems

(1) Convert each percent to a fraction

$$(a) 32\% \quad (b) 78\% \quad (c) 64\% \quad (d) 150\% \quad (e) 2\frac{1}{2}\% \quad (f) 11\frac{1}{4}\%$$

(2) Convert each percent to a decimal

$$(a) 55\% \quad (b) 72\% \quad (c) 8.5\% \quad (d) 3\% \quad (e) 125\% \quad (f) 6\frac{1}{2}\%$$

(3) Convert each fraction to a percent

$$(a) \frac{7}{10} \quad (b) \frac{2}{5} \quad (c) \frac{5}{12} \quad (d) \frac{7}{4} \quad (e) \frac{5}{8} \quad (f) 2\frac{1}{4}$$

(4) Convert each decimal to a percent

$$(a) 0.08 \quad (b) 0.56 \quad (c) 0.825 \quad (d) 0.063 \quad (e) 1.82 \quad (f) 3$$

Answers

(1) (a) $\frac{8}{25}$ (b) $\frac{39}{50}$ (c) $\frac{16}{25}$ (d) $\frac{3}{2}$ (e) $\frac{1}{40}$ (f) $\frac{9}{80}$

(2) (a) 0.55 (b) 0.72 (c) 0.085 (d) 0.03 (e) 1.25 (f) 0.065

(3) (a) 70% (b) 40% (c) $41\frac{2}{3}\%$ (d) 175% (e) $62\frac{1}{2}\%$ (f) 225%

(4) (a) 8% (b) 56% (c) 82.5% (d) 6.3% (e) 182% (f) 300%

Solving Percent Problems

$$\text{Amount} = \text{Percent} \times \text{Base}$$

$$A = P \times B$$

Amount is the part of the base

Percent is the part of the base in hundredths (always contains the percent sign)

Base is the number we are taking the percent of (always follows the word of)

Three types of problems

(1) 20% of 120 is what number or Find 20% of 120

$$P \times B = A$$

$$0.20 \times 120 = A$$

$$24 = A$$

(2) 15% of what number is 30 or 30 is 15% of what number

$$P \times B = A$$

$$0.15 \times B = 30$$

$$B = \frac{30}{.15}$$

$$B = 200$$

(3) What percent of 20 is 10 or 10 is what percent of 20

$$P \times B = A$$

$$P \times 20 = 10$$

$$P = \frac{10}{20}$$

$$P = 50\%$$

Problems

- (1) 35% of 120 is what number
- (2) What is $12\frac{1}{2}\%$ of 72
- (3) 15% of what number is 24
- (4) 28 is 40% of what number
- (5) What percent of 60 is 15
- (6) 12 is what percent of 96
- (7) 50 is what percent of 40
- (8) A meal costs \$18.80. A diner wants to leave a 15% tip. How much is the tip?
- (9) A student answers 96 questions correctly on a 120 question multiple-choice exam. What percent of the questions were answered correctly?
- (10) A person buying a home made a down payment of 20% of the purchase price. The down payment was \$36,000. What was the purchase price of the home?

Answers

- | | |
|-----------------------|----------------|
| (1) 42 | (9) 80% |
| (2) 9 | (10) \$180,000 |
| (3) 160 | |
| (4) 70 | |
| (5) 25% | |
| (6) $12\frac{1}{2}\%$ | |
| (7) 125% | |
| (8) \$2.82 | |

Ratio and Proportion

Ratio and Proportion

A ratio is the quotient of two numbers of like quantities and is of the form $\frac{a}{b}$

A ratio can be expressed as: $\frac{a}{b}$ or $a:b$ or a to b

Example: If the Red Sox win 96 games of the 162 games played. What is the ratio of wins to games played?

$$\frac{96}{162} = \frac{16}{27}$$

A rate is the ratio of unlike quantities

Example: A person earns \$140 for 8 hours of work. What is the person's hourly rate?

$$\frac{\$140}{8 \text{ hours}} = \frac{\$17.50}{1 \text{ hour}} = \frac{\$17.50}{\text{hour}}$$

Problems

(1) Write each ratio in simplest form:

- (a) \$1500 to \$250
- (b) 300 miles to 750 miles
- (c) 12 hours to 8 hours
- (d) In a town election, 850 people voted for a tax increase, 1250 people voted against the tax increase. What is the ratio of those who voted against the tax increase to those who voted for a tax increase?

(2) Write each rate in simplest form:

- (a) 400 miles to 16 gallons of gas
- (b) 48 hits to 150 times at bat
- (c) 8 cups of milk to 2 cups of flour
- (d) 25 carbohydrates to 10 ounces of food

Answers

(1)

- (a) $\frac{6}{1}$
- (b) $\frac{2}{5}$
- (c) $\frac{3}{2}$
- (d) $\frac{25}{17}$

(2)

- (a) $\frac{25 \text{ mi.}}{1 \text{ gal.}}$
- (b) $\frac{8 \text{ hits}}{25 \text{ times at bat}}$
- (c) $\frac{4 \text{ cups of milk}}{1 \text{ cup of flour}}$
- (d) $\frac{5 \text{ carbohydrates}}{2 \text{ oz.}}$

Proportion

A proportion is a statement of equality between two ratios and is of the form: $\frac{a}{b} = \frac{c}{d}$

Rule of proportions: $a \times d = b \times c$

Examples:

$$\begin{aligned} \text{(a)} \quad \frac{3}{5} &= \frac{n}{20} \\ 3 \times 20 &= 5 \times n \\ 60 &= 5n \\ \frac{60}{5} &= n \\ 12 &= n \end{aligned}$$

- (b) A scale on a map is $\frac{1}{2}$ inch to 50 miles. What is the distance between 2 cities that are 8 inches apart on the map?

Make sure the labels in the “numerators” match and the labels in the “denominators” match

$$\begin{aligned} \frac{\frac{1}{2} \text{ in.}}{50 \text{ mi.}} &= \frac{8 \text{ in.}}{n \text{ mi.}} \\ \frac{1}{2} \times n &= 50 \times 8 \\ \frac{1}{2} n &= 400 \\ n &= 400 \times 2 \\ n &= 800 \text{ mi.} \end{aligned}$$

Problems

- (1) 12 gallons of gasoline are used to travel 264 miles. How many gallons would be used to travel 462 miles?
- (2) 30 deer are tagged in a wildlife preserve. Later, 50 deer are captured and 3 are tagged. Estimate the number of deer in the preserve.
- (3) A recipe requires 3 cups of flour for 5 cups of milk. How many cups of flour are required for 3 cups of milk?

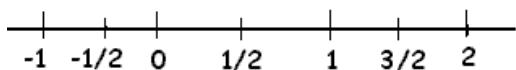
Answers

- (1) 21 gallons used
- (2) 500 deer in the preserve
- (3) $1\frac{4}{5}$ cups of flour

Signed Numbers

Signed Numbers

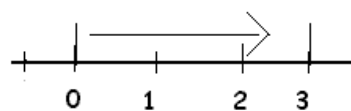
Numbers can be represented geometrically by points on a number line.



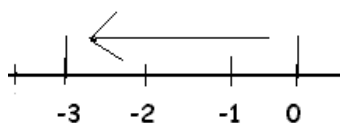
A positive number is a number $>$ (greater than) 0

A negative number is a number $<$ (less than) 0

Absolute Value: is the number of units of distance from zero to any point on the number line, and is denoted by $|a|$, read, absolute value of a



3 units of distance $|3| = 3$



3 units of distance $|-3| = 3$

$$|0| = 0$$

Addition of Numbers of Like Signs

(1) Add the absolute values of the numbers

(2) The sign of the answer is the common sign

Examples: Add

$$(a) 15 + 17 = +(|15| + |17|) = +(15 + 17) = 32$$

$$(b) (-8) + (-10) = -(|-8| + |-10|) = -(8 + 10) = -18$$

$$(c) (-3) + (-5) + (-2) = -(|-3| + |-5| + |-2|) = -(3 + 5 + 2) = -10$$

$$(d) \left(-\frac{1}{4}\right) + \left(-\frac{1}{6}\right) = \left(-\frac{3}{12}\right) + \left(-\frac{2}{12}\right) = -\left(\left|-\frac{3}{12}\right| + \left|-\frac{2}{12}\right|\right) = -\left(\frac{3}{12} + \frac{2}{12}\right) = -\frac{5}{12}$$

Addition of Numbers of Unlike Signs

(1) Subtract the smaller absolute value from the larger absolute value

(2) The sign of the answer is the sign of the number with the larger absolute value

Examples:

$$(a) 10 + (-25) = -(25 - 10)$$

$$(b) -12 + 18 = +(18 - 12) = 6$$

$$(c) \frac{2}{5} + \left(-\frac{3}{7}\right) = \frac{14}{35} + \left(-\frac{15}{35}\right) = -\left(\frac{15}{35} - \frac{14}{35}\right) = -\frac{1}{35}$$

$$(d) -3 + 5 + (-8) + 7 + 6 = 5 + 7 + 6 + (-3) + (-8) = 18 + (-11) = 18 - 11 = 7$$

(You can either combine two numbers at a time from left to right *or*
Group and combine all positives, group and combine all negatives, then
combine the “positive” and “negative” result)

Problems

Add

$$(1) -15 + 6$$

$$(5) \frac{3}{4} + \left(-\frac{2}{3}\right)$$

$$(2) -20 + 32$$

$$(6) \left(-\frac{2}{5}\right) + \left(-\frac{1}{4}\right)$$

$$(3) -16 + (-14)$$

$$(7) 9 + (-5) + (-8) + 4 + 3$$

$$(4) -17 + (-9)$$

$$(8) -12 + 15 + (-11) + (-4) + 2$$

Answers

$$(1) -9$$

$$(5) \frac{1}{12}$$

$$(2) 12$$

$$(6) -\frac{13}{20}$$

$$(3) -30$$

$$(7) 3$$

$$(4) -26$$

$$(8) -10$$

Subtraction of Signed Numbers

$$12 - 8 = 4 \text{ and } 12 + (-8) = 4$$

$$\text{Thus } 12 - 8 = 12 + (-8)$$

We then have a definition for subtraction: $a - b = a + (-b)$, where a and b are any numbers

To Subtract Two Signed Numbers

- (1) Change the operation from subtraction to addition and change the sign of the second number
- (2) Follow the methods of adding signed numbers

Examples: Subtract

$$(a) 9 - 10 = 9 + (-10) = -1$$

$$(b) 5 - (-7) = 5 + (7) = 12$$

$$(c) -4 - \left(-\frac{1}{2}\right) = -4 + \frac{1}{2} = -\frac{8}{2} + \frac{1}{2} = -\frac{7}{2} = -3\frac{1}{2}$$

$$(d) -6 - 12 = -6 + (-12) = -18$$

Problems

Subtract

$$(1) -8 - (-12)$$

$$(4) -\frac{5}{6} - \left(-\frac{2}{3}\right)$$

$$(2) 6 - 14$$

$$(5) 12 - (-17)$$

$$(3) -3 - 9$$

$$(6) 2 - \frac{5}{2}$$

Answers

$$(1) 4$$

$$(4) -\frac{1}{6}$$

$$(2) -8$$

$$(5) 29$$

$$(3) -12$$

$$(6) -\frac{1}{2}$$

Multiplication of Signed Numbers

Using the definition of multiplication:

$$2(-4) = (-4) + (-4) = -8$$

$$4(-3) = (-3) + (-3) + (-3) + (-3) = -12$$

When we multiply a positive number and a negative number, we get a negative product:

$$2(-3) = -6$$

$$1(-3) = -3$$

$$0(-3) = 0$$

$$(-1)(-3) = 3$$

Pattern suggests that when we multiply a negative number with a negative number, we get a positive product.

To Multiply Two Signed Numbers of Like Sign

(1) Multiply their absolute values

(2) Product is positive

Examples:

$$(a) (-5)(-6) = +(5 \times 6) = 30$$

$$(b) \left(-\frac{3}{5}\right)\left(-\frac{25}{24}\right) = +\left(\frac{3}{5} \times \frac{25}{24}\right) = \left(\frac{1}{1} \times \frac{5}{8}\right) = \frac{5}{8}$$

$$(c) (-6)(5)(2)(-1) = (-6)(-1)(5)(2) = (6)(10) = 60$$

To Multiply Two Signed Numbers of Unlike Signs

(1) Multiply their absolute values

(2) Product is negative

Examples:

$$(a) (-10)(3) = -(10 \times 3) = -30$$

$$(b) \left(\frac{7}{10}\right)\left(-\frac{5}{14}\right) = -\left(\frac{7}{10} \times \frac{5}{14}\right) = -\left(\frac{1}{2} \times \frac{1}{2}\right) = -\frac{1}{4}$$

Problems:

Multiply

(1) $(-8)(-12)$

(4) $\left(\frac{9}{10}\right)\left(-\frac{5}{6}\right)$

(2) $(-5)(9)$

(5) $(-6)(10)\left(-\frac{1}{2}\right)(2)$

(3) $\left(-\frac{7}{12}\right)\left(-\frac{9}{14}\right)$

(6) $(-4)(-3)(5)(-1)$

Answers

(1) 96

(4) $-\frac{3}{4}$

(2) -45

(5) 60

(3) $\frac{3}{8}$

(6) -60

Division of Signed Numbers

$$\frac{35}{5} = 7 \text{ because } 7 \times 5 = 35$$

$$\frac{-15}{-3} = 5 \text{ because } 5(-3) = -15$$

$$\frac{-28}{7} = -4 \text{ because } (-4)(7) = -28$$

Thus: To Divide Two Numbers of Like Signs

(1) Divide the absolute values of the numbers

(2) Quotient is positive

Examples:

$$(a) \frac{-20}{-10} = \frac{|-20|}{|-10|} = \frac{20}{10} = 2$$

$$(b) \left(-\frac{5}{8}\right) \div \left(-\frac{25}{16}\right) = \frac{|-5|}{|8|} \div \frac{|-25|}{|16|} = \frac{5}{8} \div \frac{25}{16} = \frac{5}{8} \times \frac{16}{25} = \frac{1}{1} \times \frac{2}{5} = \frac{2}{5}$$

To Divide Two Numbers of Unlike Signs

- (1) Divide the absolute values of the numbers
- (2) Quotient is negative

Examples:

$$(a) \frac{-40}{8} = -\frac{40}{8} = -5$$

$$(b) \left(-\frac{2}{3}\right) \div \left(\frac{8}{9}\right) = \left(\frac{2}{3} \div \frac{8}{9}\right) = \left(\frac{2}{3} \times \frac{9}{8}\right) \left(\frac{1}{1} \times \frac{3}{4}\right) = -\frac{3}{4}$$

Problems

Divide

$$(1) \frac{-35}{-7}$$

$$(5) \left(\frac{8}{9}\right) \div \left(-\frac{4}{3}\right)$$

$$(2) \frac{-50}{10}$$

$$(6) (-8.2) \div (-4.1)$$

$$(3) \frac{56}{-8}$$

$$(7) (-0.0636) \div (5.3)$$

$$(4) \left(-\frac{1}{2}\right) \div \left(-\frac{5}{2}\right)$$

Answers

$$(1) 5$$

$$(5) -\frac{2}{3}$$

$$(2) -5$$

$$(6) 2$$

$$(3) -7$$

$$(7) -0.012$$

$$(4) \frac{1}{5}$$

Exponents

Exponents

When a number is used as a repeated factor: $4 \times 4 \times 4$, it can be written in exponential form, 4^3 .

4 is the base: the number that is being multiplied

3 is the exponent: tells us how many times the base is being multiplied by itself

Examples:

$$(a) 2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

$$(b) (-10)^2 = (-10)(-10) = 100$$

Since -10 is inside the (), -10 is the base

$$(c) -2^4 = -1(2^4) = -1(2 \times 2 \times 2 \times 2) = -16$$

Since there is no (), just the 2 is the base...not the “-“ sign

$$(d) 5^1 = 5$$

Laws of Exponents

(1) Product Rule

$$2^3 \times 2^2 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$$

This example illustrates:

$$a^m \times a^n = a^{m+n}$$

(Since bases are the same...keep the base and add the exponents)

Examples:

$$(a) 5^4 \times 5^3 = 5^{4+3} = 5^7 = 78125$$

$$(b) (-3)^2 (-3) = (-3)^{2+1} = (-3)^3 = -27$$

(2) Quotient Rule

$$\frac{5^5}{5^2} = \frac{5 \times 5 \times 5 \times 5 \times 5}{5 \times 5} = \frac{5 \times 5 \times 5 \times 1 \times 1}{1 \times 1} = 5^3 = 125$$

This example illustrates:

$$\frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$$

(Since the bases are the same...keep the base and subtract the exponents)

Examples:

$$(a) \frac{6^5}{6^3} = 6^{5-3} = 6^2 = 36$$

$$(b) \frac{4^5}{4^4} = 4^{5-4} = 4^1 = 4$$

(3) Zero Exponent

$$\frac{3^2}{3^2} = 3^{2-2} = 3^0 \quad \text{or} \quad \frac{3^2}{3^2} = \frac{9}{9} = 1$$

Thus $3^0 = 1$

This example illustrates

$$a^0 = 1, \quad a \neq 0$$

Examples:

(a) $10^0 = 1$

(b) $(-6)^0 = 1$

(c) $-5^0 = -1 \times 5^0 = -1$

(4) Negative Exponent

$$\frac{2^2}{2^3} = 2^{2-3} = 2^{-1} \quad \text{or} \quad \frac{2^2}{2^3} = \frac{2 \times 2}{2 \times 2 \times 2} = \frac{1 \times 1}{1 \times 1 \times 2} = \frac{1}{2}$$

Thus $2^{-1} = \frac{1}{2}$

This example illustrates

$$a^{-n} = \frac{1}{a^n}, \quad a \neq 0$$

(Negative exponent shifts position of its base: numerator \Leftrightarrow denominator)

Examples:

(a) $3^{-4} = \frac{1}{3^4}$

(b) $\frac{1}{5^{-4}} = 5^4$

(5) Raising a Product to a Power

$$(5y)^3 = (5y)(5y)(5y) = (5 \times 5 \times 5)(y \times y \times y) = 5^3 y^3$$

This example illustrates

$$(ab)^n = a^n b^n$$

Examples:

(a) $(2xy)^3 = 2^3 x^3 y^3 = 8x^3 y^3$

(b) $(-3a)^4 = (-3)^4 a^4 = 81a^4$

(6) Raising a Power to a Power

$$(2^3)^2 = 2^3 \times 2^3 = 2^6$$

This example illustrates

$$(a^m)^n = a^{mn}$$

(When raising to a power: Keep base...multiply exponents)

Examples:

$$(a) (3^2)^4 = 3^{2 \times 4} = 3^8 = 6561$$

$$(b) (2a^3b^2)^3 = 2^3 (a^3)^3 (b^2)^3 = 2^3 a^9 b^6 = 8a^9 b^6$$

(7) Raising a Fraction to a Power

$$\left(\frac{4}{5}\right)^3 = \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \frac{4 \times 4 \times 4}{5 \times 5 \times 5} = \frac{4^3}{5^3} = \frac{64}{125}$$

This example illustrates

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad b \neq 0$$

Examples:

$$(a) \left(\frac{2a}{3b}\right)^5 = \frac{(2a)^5}{(3b)^5} = \frac{2^5 a^5}{3^5 b^5} = \frac{32a^5}{243b^5}$$

$$(b) \left(\frac{-5a^2}{6b}\right)^2 = \frac{(-5a^2)^2}{(6b)^2} = \frac{(-5)^2 a^4}{6^2 b^2} = \frac{25a^4}{36b^2}$$

Problems

Simplify

(1) $2^4 \times 2^3$

(6) -4^0

(2) $10 \times 10^2 \times 10^3$

(7) $(5a^2b)^3$

(3) $\frac{5^6}{5^4}$

(8) $(-2a^2b)$

(4) $\frac{2^3}{2^5}$

(9) $\left(\frac{3a^3}{4b^2}\right)^2$

(5) 12^0

(10) $\left(\frac{-5a}{2b}\right)^3$

Answers

(1) $2^7 = 128$

(2) $10^6 = 1,000,000$

(3) $5^2 = 25$

(4) $\frac{1}{2^2} = \frac{1}{4}$

(5) 1

(6) -1

(7) $5^3 a^6 b^3 = 125a^6b^3$

(8) $(-2)^2 a^4 b^2 = 4a^4b^2$

(9) $\frac{3^2 a^6}{4^2 b^4} = \frac{9a^6}{16b^4}$

(10) $\frac{(-5)^3 a^3}{2^3 b^3} = \frac{-125a^3}{8b^3}$

Algebraic Expressions

Algebraic Expressions

Term: a number, variable, or a product or quotient of numbers and variables

Examples: 5, x, -3xy, $\frac{x}{5}$

Numbered part of the term is called the numerical coefficient or coefficient. For the term -3xy, 3 is the coefficient

Like Terms: variable parts of the terms are exactly the same; i.e. same variable, same exponent

Examples:

(a) 4ab and -3ab

(b) $2x^2y$ and $5x^2y$

Unlike Terms: variable parts of the terms differ; i.e. different variables or the same variable
With different exponents

Examples:

(a) 5a and 7b

(b) $3a^2b$ and -10ab

Evaluating Algebraic Expressions

Order of Operations

- (1) Perform operations inside the symbols of grouping-parentheses, brackets, fraction bars, etc.
- (2) Evaluate powers of numbers
- (3) Do all multiplications and divisions in order from left to right
- (4) Do all additions and subtractions in order from left to right

Examples:

(a) $5^2 - 6 \times 5 + 10 \div 2 - 4$
 $25 - 6 \times 5 + 10 \div 2 - 4$
 $25 - 30 + 5 - 4$
 $-5 + 5 - 4$
 $0 - 4$
 -4

$$\begin{aligned}
 \text{(b)} \quad & 4(3^2 + 5 \times 2) - \frac{10^2 + 8}{2^3 + 1} \\
 & 4(9 + 10) - \frac{100 + 8}{8 + 1} \\
 & 4(19) - \frac{108}{9} \\
 & 76 - 12 \\
 & 64
 \end{aligned}$$

To evaluate any algebraic expression replace the variable by its numerical value and follow the order of operations.

Examples: Evaluate each expression for the given values of the variables

$$\begin{aligned}
 \text{(a)} \quad & 3x^2 - 5xy + y^2 && \text{Let } x = 2, y = -3 \\
 & 3(2)^2 - 5(2)(-3) + (-3)^2 \\
 & 3(4) - 5(2)(-3) + 9 \\
 & 12 + 30 + 9 \\
 & 51
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & \frac{2x + 5y}{x^2 - 4xy} && \text{Let } x = -2, y = 4 \\
 & \frac{2(-2) + 5(4)}{(-2)^2 - 4(-2)(4)} \\
 & \frac{-4 + 20}{4 + 32} \\
 & \frac{16}{36} \\
 & \frac{4}{9}
 \end{aligned}$$

Polynomials

Monomial: an algebraic expression containing one term: ex. $6x^2$

Binomial: an algebraic expression containing two terms: ex. $3x - 5$

Trinomial: an algebraic expression containing three terms: ex. $x^2 - 5x + 6$

Polynomial: an algebraic expression containing more than three terms:

$$\text{ex. } 5x^3 + 2x^2y - xy^2 + y^3$$

Addition and Subtraction of Algebraic Expressions

To add or subtract algebraic expressions: we add or subtract like terms by combining the numerical coefficients and attaching the variable part of the term to the answer.

Examples:

$$(a) 5x^2 - 10x^2 + 3x^2 = -2x^2$$

$$(b) \begin{aligned} &4x^2y + 2y^2 - 3x^2y + 5y^2 \\ &4x^2y - 3x^2y + 2y^2 + 5y^2 \\ &x^2y + 7y^2 \end{aligned}$$

$$(c) \begin{aligned} &(3a^2 + 6b^2 - 5c^2) + (7a^2 - 8b^2 - c^2) \\ &(3a^2 + 7a^2) + (6b^2 - 8b^2) + (-5c^2 - c^2) \\ &10a^2 - 2b^2 - 6c^2 \end{aligned}$$

$$(d) \begin{aligned} &(9a - 5b) - (6a + 2b) \quad (\text{Remember the definition of Subtraction}) \\ &9a - 5b - 6a - 2b \\ &(9a - 6a) + (-5b - 2b) \\ &3a - 7b \end{aligned}$$

$$(e) \begin{aligned} &(x^2 + 6x + 8) - (3x^2 - 2x + 8) \\ &x^2 + 6x + 8 - 3x^2 + 2x - 8 \\ &(x^2 - 3x^2) + (6x + 2x) + (8 - 8) \\ &-2x^2 + 8x \end{aligned}$$

Problems

Combine

$$(1) 15x^2 + 12x^2 - 7x^2$$

$$(2) 4x^3 + 5x - 8x^3 - 2x + 3$$

$$(3) (3a^2b - 9ab + 2) + (7a^2b + 7ab - 4)$$

$$(4) (7a^3 - 4a^2 - 5) - (-2a^3 + 5a^2 - 6)$$

$$(5) (-10a^3b + 12a^2b^2 - 6ab^3) - (8a^3b + 6a^2b^2 - 9ab^3)$$

Answers

- (1) $20x^2$
- (2) $-4x^3 + 3x + 3$
- (3) $10a^2b - 2ab - 2$
- (4) $9a^3 - 9a^2 + 1$
- (5) $-18a^3b + 6a^2b^2 + 3ab^3$

Multiplication of Algebraic Expressions

(1) Multiplying Two or More Monomials

Follow the laws of signed numbers and the laws of exponents

Examples:

$$(a) (5x^2y)(-4xy) = 5(-4)(x^2 \times x)(y \times y) = -20x^3y^2$$

$$(b) (2ab)(-3ab^2)(4a) = (2)(-3)(4)(a \times a \times a)(b \times b^2) = -24a^3b^3$$

(2) Multiplying a Monomial and a Polynomial

Multiply each term of the polynomial by the monomial

Examples:

$$(a) 5(x^2 + 4x - 2) = 5x^2 + 5(4x) + 5(-2) = 5x^2 + 20x - 10$$

$$(b) 3ax(2x^2 - 3ax + 4a^2) = 3ax(2x^2) + 3ax(-3ax) + 3ax(4a^2) = \\ = 6ax^3 - 9a^2x^2 + 12a^3x$$

$$(c) 5x(2x - 3) - 3(x^2 + 4) = 5x(2x) + 5x(-3) - 3(x^2) - 3(4) = \\ = 10x^2 - 15x - 3x^2 - 12 = 7x^2 - 15x - 12$$

(3) Multiplying a Polynomial and a Polynomial

Multiply each term of one polynomial by each term of the other polynomial and combine the like terms.

Examples:

$$(a) (3x + 5)(x^2 - x + 2) = 3x(x^2) + 3x(-x) + 3x(2) + 5(x^2) + 5(-x) + 5(2) \\ = 3x^3 - 3x^2 + 6x + 5x^2 - 5x + 10 = 3x^3 + 2x^2 + x + 10$$

$$(b) (2a - b)(4a + 5b) = 2a(4a) + 2a(5b) - b(4a) - b(5b) \\ = 8a^2 + 10ab - 4ab - 5b^2 = 8a^2 + 6ab - 5b^2$$

$$(c) (x+5)(x-5) = x(x) + x(-5) + 5(x) + 5(-5) = \\ = x^2 - 5x + 5x - 25 = x^2 - 25$$

$$\text{Observe: } (a+b)(a-b) = a^2 - b^2$$

$$(d) (3x+4)(3x-4) = (3x)^2 - 4^2 = 9x^2 - 16$$

$$(e) (x+6)^2 = (x+6)(x+6) = x(x) + x(6) + 6(x) + 6(6) = \\ = x^2 + 6x + 6x + 36 = x^2 + 12x + 36$$

$$\text{Observe: } (a+b)^2 = a^2 + 2ab + b^2$$

$$(f) (2a+3b)^2 = (2a)^2 + 2(2a)(3b) + (3b)^2 = 4a^2 + 12ab + 9b^2$$

$$(g) (3x-5y)^2 = (3x-5y)(3x-5y) = \\ = 3x(3x) + 3x(-5y) - 5y(3x) - 5y(-5y) = 9x^2 - 15xy - 15xy + 25y^2 \\ = 9x^2 - 30xy + 25y^2$$

$$\text{Observe: } (a-b)^2 = a^2 - 2ab + b^2$$

$$(h) (a-3b)^2 = (a)^2 - 2(a)(3b) + (3b)^2 = a^2 - 6ab + 9b^2$$

Problems

Perform the indicated operations

$$(1) (3a^2b)(7ab^3)$$

$$(6) (2a-b)(a^2+4ab-b^2)$$

$$(2) (5a^3)(-2a^2b)(3b^2)$$

$$(7) (3x+5)(2x-3)$$

$$(3) 6(a^2+4ab-2b^2)$$

$$(8) (7x+5y)(7x-5y)$$

$$(4) -3a(-2a^2+5)$$

$$(9) (5x+3)^2$$

$$(5) 4a(a^2+2a-1) - 2(a^3-2a+5)$$

$$(10) (2x-7y)^2$$

Answers

(1) $21a^3b^4$

(6) $2a^3 + 7a^2b - 6ab^2 + b^3$

(2) $-30a^5b^3$

(7) $6x^2 + x - 15$

(3) $6a^2 + 24ab - 12b^2$

(8) $49x^2 - 25y^2$

(4) $6a^3 - 15a$

(9) $25x^2 + 30x + 9$

(5) $2a^3 + 8a^2 - 10$

(10) $4x^2 - 28xy + 49y^2$

Division of Algebraic Expressions

(1) Division of Two Monomials

Follow the laws of signed numbers and the laws of exponents

Examples:

(a) $\frac{10x^3y^2}{5x^2y} = \frac{10}{5} \times \frac{x^3}{x^2} \times \frac{y^2}{y} = 2xy$

(b) $\frac{-20x^5y}{15x^3y^2} = \frac{-20}{15} \times \frac{x^5}{x^3} \times \frac{y}{y^2} = \frac{-5x^2}{3y}$

(2) Division of a Polynomial by a Monomial

Divide each term of the polynomial by the monomial

Example: $\frac{10x^3y^2 - 15x^2y^2 + 5xy^2}{5xy} = \frac{10x^3y^2}{5xy} - \frac{15x^2y^2}{5xy} + \frac{5xy^2}{5xy} = 2x^2y - 3xy + y$

(3) Division of One Polynomial by Another

- (1) Arrange the dividend and divisor in descending powers of the variable
- (2) Divide the first term of the dividend by the first term of the divisor. You get the first (next) term of the quotient
- (3) Multiply the divisor by the new term you just put in the quotient and subtract this product from the dividend
- (4) Repeat the process beginning with step (2)

Examples:

(a) $(6x^2 - x - 15) \div (3x - 5)$

$$\begin{array}{r} 2x+3 \\ 3x-5 \overline{) 6x^2 - x - 15} \\ \underline{\mp 6x^2 \pm 10x} \\ 9x - 15 \\ \underline{\mp 9x \pm 15} \\ 0 \end{array}$$

(b) $(3x^3 + 2x^2 - 5) \div (x + 2)$

$$\begin{array}{r} 3x^2 - 4x + 8 \\ x+2 \overline{) 3x^3 + 2x^2 + 0x - 5} \text{ (notice insertion of place saver } 0x) \\ \underline{\mp 3x^3 \mp 6x^2} \\ -4x^2 + 0x - 5 \\ \underline{\pm 4x^2 \pm 8x} \\ 8x - 5 \\ \underline{\mp 8x \mp 16} \\ -21 \end{array}$$

Problems

Divide

(1) $\frac{24a^3b^4c}{12a^2b^2c}$

(4) $\frac{-24a^4b^3 + 18a^3b^2 - 12a^2b}{12a^3b}$

(2) $\frac{-30a^4b^2}{15a^6b^4}$

(5) $(2x^2 - 3x - 20) \div (2x + 5)$

(3) $\frac{18a^3b + 12a^2b - 6ab}{6ab}$

(6) $(4x^3 + 2x^2 + 3) \div (x - 1)$

Answers

(1) $2ab^2$

(4) $-2ab^2 + \frac{3b}{2} - \frac{1}{a}$

(2) $\frac{-2}{a^2b^2}$

(5) $x - 4$

(3) $3a^2 + 2a - 1$

(6) $4x^2 + 6x + 6 \text{ R}9$

Factoring

Factoring

To factor an algebraic expression we write the algebraic expression as a product of its factors

- (1) Finding the greatest common factor: the largest number that divides evenly into each term

Examples:

(a) Factor: $5x^2 - 10xy$
 $5x(x - 2y)$

$5x$ is the greatest common factor
Divide $5x^2 - 10xy$ by $5x$

(b) Factor: $8x^3y + 16x^4y^2 - 24x^5y^3$
 $8x^3y(1 + 2xy - 3x^2y^2)$

$8x^3y$ is the greatest common factor

(c) Factor: $5x(y - 2) + 3(y - 2)$
 $(y - 2)(5x + 3)$

$y - 2$ is the greatest common factor

In factoring any algebraic expression, the first step is always to look for a Greatest common factor.

- (2) The Difference of Two Squares

Noting that $(a + b)(a - b) = a^2 - b^2$. A binomial of the form $a^2 - b^2$ has for its factors $(a + b)(a - b)$

Examples:

(a) Factor: $x^2 - 25$
 $x^2 - 25 = x^2 - 5^2 = (x + 5)(x - 5)$

(b) Factor: $9x^2 - 64y^2$
 $9x^2 - 64y^2 = (3x)^2 - (8y)^2 = (3x + 8y)(3x - 8y)$

(c) Factor: $5y^2 - 20$
 $5y^2 - 20 = 5(y^2 - 4) = 5(y + 2)(y - 2)$

(d) Factor: $a^2 + b^2$
 $a^2 + b^2$ is prime. Only factors are 1 and $a^2 + b^2$

(3) Trinomials of the Form $x^2 + bx + c$

Noting that $(x+m)(x+n) = x^2 + (m+n)x + mn$: we factor trinomials of the form $x^2 + bx + c$ by:

- (1) Finding 2 numbers, m and n , whose product is c and whose sum is b
- (2) Writing them in the form $(x+m)(x+n)$

Examples:

(a) Factor: $x^2 + 5x + 6$

Two numbers whose product is 6 and sum is 5: 3 and 2

$$x^2 + 5x + 6 = (x+3)(x+2)$$

(b) Factor: $x^2 - 8x + 12$

Two numbers whose product is 12 and sum is -8: -6 and -2

$$x^2 - 8x + 12 = (x-6)(x-2)$$

(c) Factor: $x^2 - 3x - 54$

Two numbers whose product is -54 and sum is -3: -9 and 6

$$x^2 - 3x - 54 = (x-9)(x+6)$$

(4) Trinomials of the form $ax^2 + bx + c, a \neq 1$

Noting that $(rx+m)(sx+n) = rsx^2 + (rn+sm)x + mn$: We factor trinomials of the form $ax^2 + bx + c, a \neq 1$ by:

- (1) Writing all pairs of factors of a
- (2) Write all pairs of factors of c
- (3) Try different combinations of factors of a and c so that the sum of the products of the inner terms and outer terms is bx

Examples:

(a) Factor: $4x^2 + 7x + 3$

Step 1: Factors of 4: (1)(4), (2)(2)

Step 2: Factors of 3: (1)(3), (-1)(-3)

Since both 3 and 7 are positive we only use the positive factors

Step 3:

$$(4x+3)(x+1)$$

$$\begin{array}{c} \text{---} \\ 4x \quad 3x \\ \text{---} \\ 4x \quad 7x \end{array}$$

Answers

(1) $6ab(a+2b)$

(2) $5xy^2(2-3y)$

(3) $(x+4)(x+3)$

(4) $(x-9)(x-2)$

(5) $(x+9)(x-7)$

(6) $4a(a^2-2a-8) = 4a(a-4)(a+2)$

(7) $(4a+5)(a+1)$

(8) $(3a-2)(2a-3)$

(9) $(3a+5)(a-3)$

(10) $(2a+7)(2a-3)$

(11) $(3x+5)(3x-5)$

(12) $(4x+y)(4x-y)$

Equations and Inequalities

Equations and Inequalities

Equation

An equation is a statement of equality between two algebraic expressions

Examples:

(a) $x + 10 = 15$

(b) $3x + 5 = 2x - 3$

(c) $4(x + 1) = 5x + 9$

(d) $\frac{x}{3} + 2 = \frac{x}{2} + \frac{1}{a}$

A solution or root to an equation is a value of the variable that makes the equation a true statement

Example: $\frac{x}{3} + 2 = \frac{x}{6} + 1$, $x = -6$ is a solution

$$\frac{-6}{3} + 2 = \frac{-6}{6} + 1$$

$$-2 + 2 = -1 + 1$$

$$0 = 0$$

Linear Equations (degree of variable = 1)

A linear equation is an equation of the form: $ax + b = 0$

To solve a linear equation, isolate the variable on one side of the equation and the non-variable terms on the other side. We do this by using the following properties of equality:

Addition Property of Equality

If $a = b$ and c is any real number then $a + c = b + c$

Subtraction Property of Equality

If $a = b$ and c is any real number then $a - c = b - c$

Multiplication Property of Equality

If $a = b$ and c is any real number, $c \neq 0$, $ac = bc$

Division Property of Equality

If $a = b$ and c is any real number, $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$

Examples:

(a) Solve: $5x+12=17$

$$5x+12=17$$

$$5x+12\boxed{-12}=17\boxed{-12} \quad \text{Subtraction Property}$$

$$\frac{5x}{\boxed{5}}=\frac{5}{\boxed{5}}$$

Division Property

$$x=1$$

(b) Solve: $3(x-2)=5x+8$

$$3x-6=5x+8 \quad \text{Remove Parentheses}$$

$$3x-6\boxed{+6}=5x+8\boxed{+6} \quad \text{Addition Property}$$

$$3x=5x+14$$

$$3x\boxed{-5x}=5x\boxed{-5x}+14 \quad \text{Subtraction Property}$$

$$-2x=14$$

$$\frac{-2x}{\boxed{-2}}=\frac{14}{\boxed{-2}}$$

Division Property

$$x=-7$$

(c) Solve: $\frac{x}{3}+3=\frac{x}{4}+2$

Multiply both sides of the equation by the LCD (12)

$$\boxed{12}\left(\frac{x}{3}+3\right)=\boxed{12}\left(\frac{x}{4}+2\right) \quad \text{Multiplication Property}$$

$$4x+36=3x+24$$

$$4x+36\boxed{-36}=3x+24\boxed{-36} \quad \text{Subtraction Property}$$

$$4x=3x-12$$

$$4x\boxed{-3x}=3x\boxed{-3x}-12 \quad \text{Subtraction Property}$$

$$x=-12$$

(d) Solve: $2x-3.4=0.4x-1.8$

$$2x-3.4\boxed{+3.4}=0.4x-1.8\boxed{+3.4} \quad \text{Addition Property}$$

$$2x=0.4x+1.6$$

$$2x\boxed{-0.4x}=0.4x\boxed{-0.4x}+1.6 \quad \text{Subtraction Property}$$

$$1.6x=1.6$$

$$\frac{1.6x}{\boxed{1.6}}=\frac{1.6}{\boxed{1.6}}$$

Division Property

$$x=1$$

(e) Solve: $3x + 2x - 6 = 4(x + 3) + 2$

$3x + 2x - 6 = 4x + 12 + 2$ Remove Parentheses

$5x - 6 = 4x + 14$ Combine Like Terms

$5x - 6 \boxed{+6} = 4x + 14 \boxed{+6}$ Addition Property

$5x = 4x + 20$

$5x - 4x = 4x \boxed{-4x} + 20$ Subtraction Property

$x = 20$

Problems

Solve

(1) $4x - 8 = 12$

(7) $\frac{2x}{3} + \frac{3}{4} = \frac{x}{2} + \frac{1}{4}$

(2) $3x + 7 = 10$

(8) $\frac{x}{5} - 1 = \frac{x}{3} + \frac{1}{3}$

(3) $5x + 12 = 3x - 6$

(9) $3.5x + 1.25 = 1.1x - 1.15$

(4) $7x - 4 = 2x + 16$

(10) $0.75x - 2.6 = 1.25x + 2.4$

(5) $5(x - 2) = 3x - 6$

(11) $6x + 10 - 4x = 5x - 8x - 5$

(6) $5 = 2(x + 4) - 1$

(12) $4 + 3(x - 5) = -4(x + 3) - 6$

Answers

(1) $x = 5$

(7) $x = -3$

(2) $x = 1$

(8) $x = -10$

(3) $x = -9$

(9) $x = -1$

(4) $x = 4$

(10) $x = -10$

(5) $x = 2$

(11) $x = -3$

(6) $x = -1$

(12) $x = -1$

Word Problems

- (1) Read the problem until it is understandable
- (2) Determine what quantity you are trying to find
- (3) Represent the quantity by a variable
- (4) Write the problem as an equation and solve the equation for the unknown variable

Examples:

- (a) Five more than twice a number is 31. What is the number?

Let n = the number

$$2n + 5 = 31$$

$$2n + 5 - 5 = 31 - 5$$

$$2n = 26$$

$$\frac{2n}{2} = \frac{26}{2}$$

$$n = 13$$

- (b) 7 times a number decreased by 5 is 7 more than 3 times the number. What is the number?

Let n = the number

$$7n - 5 = 3n + 7$$

$$7n - 5 + 5 = 3n + 7 + 5$$

$$7n = 3n + 12$$

$$7n - 3n = 3n - 3n + 12$$

$$4n = 12$$

$$\frac{4n}{4} = \frac{12}{4}$$

$$n = 3$$

- (c) The length of a rectangle is 3 feet less than twice the width. The perimeter is 30 feet. Find the length and width of the rectangle.

Let n = the width

$2n - 3$ = the length

Perimeter of a Rectangle = $2l + 2w$

$$30 = 2(2n - 3) + 2n$$

$$30 = 4n - 6 + 2n$$

$$30 = 6n - 6$$

$$30 + 6 = 6n - 6 + 6$$

$$36 = 6n$$

$$\frac{36}{6} = \frac{6n}{6}$$

$$n = 6$$

$$2n - 3 = 2(6) - 3 = 9$$

Width = 6 feet

Length = 9 feet

Problems

- (1) Four times a number decreased by 6 is 3 more than the number. What is the number?
- (2) A television set costs \$300 more than a DVD player. The total cost of the television set and DVD player is \$650. What is the cost of the television set?
- (3) The perimeter of a rectangle is 70 feet. The length is 5 feet more than the width. What are the length and width of the rectangle?
- (4) A garden is surrounded by 88 feet of fence. The length is 1 foot less than twice the width. What are the length and width of the garden?

Answers

- (1) 3
- (2) \$475
- (3) width = 15 feet
length = 20 feet
- (4) width = 15 feet
length = 29 feet

Inequalities

An inequality is a statement connecting algebraic expressions by an inequality symbol

Symbols of Inequality

$<$: less than

\leq : less than or equal to

$>$: greater than

\geq : greater than or equal to

Examples:

(a) $3x + 1 > 10$

(b) $x + 3 < -2x - 9$

(c) $3x + 5 \leq x + 7$

(d) $-4x - 1 \leq 2x + 11$

To solve a linear inequality, isolate the variable on one side of the inequality and the non-variable term on the other side. We do this by using the following properties of inequalities. The properties hold true for: $<$, \leq , $>$, \geq .

Addition Property of Inequality

If $a > b$ and c is any real number $a + c > b + c$

Subtraction Property of Inequality

If $a > b$ and c is any real number $a - c > b - c$

If we multiply or divide both sides of an inequality by a negative number, the sense of the inequality changes.

Multiplication and Division Properties of Inequality

(1) If $a > b$ and $c < 0$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

(2) If $a > b$ and $c > 0$ then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

Examples:

(a) Solve: $3x+5 > x+3$

$$3x+5 > x+3$$

$$3x+5 \boxed{-5} > x+3 \boxed{-5} \text{ Subtraction Property}$$

$$2x > -2$$

$$\frac{2x}{\boxed{2}} > \frac{-2}{\boxed{2}} \text{ Division Property}$$

$$x > -1$$

(b) Solve: $2(x-5) \leq 4x+6$

$$2(x-5) \leq 4x+6 \text{ Remove Parentheses}$$

$$2x-10 \boxed{+10} \leq 4x+6 \boxed{+10} \text{ Addition Property}$$

$$2x \leq 4x+16$$

$$2x \boxed{-4x} \leq 4x \boxed{-4x} +16 \text{ Subtraction Property}$$

$$-2x \leq 16$$

$$\frac{-2x}{\boxed{-2}} \leq \frac{16}{\boxed{-2}}$$

Division Property (dividing by a negative number)

$$x \geq -8$$

(c) Solve: $\frac{x}{4} - \frac{1}{2} \geq x-2$

$$\frac{x}{4} - \frac{1}{2} \geq x-2$$

$$\boxed{4} \left(\frac{x}{4} - \frac{1}{2} \right) \geq \boxed{4} (x-2) \text{ Multiplication Property}$$

$$x-2 \geq 4x-8$$

$$x-2 \boxed{+2} \geq 4x-8 \boxed{+2} \text{ Addition Property}$$

$$x \geq 4x-6$$

$$x \boxed{-4x} \geq 4x \boxed{-4x} -6 \text{ Subtraction Property}$$

$$-3x \geq -6$$

$$\frac{-3x}{\boxed{-3}} \leq \frac{-6}{\boxed{-3}}$$

Division Property (dividing by a negative number)

$$x \leq 2$$

Problems

Solve

(1) $5x - 6 \geq 3x + 4$

(4) $-4(2x + 1) > 5 - 7x$

(2) $2x + 9 < 4x + 5$

(5) $\frac{x}{3} + \frac{1}{2} \leq \frac{x}{4} + \frac{1}{3}$

(3) $5(x + 2) \leq 3x - 2$

(6) $\frac{x}{5} - 1 \geq \frac{x}{4} - \frac{1}{2}$

Answers

(1) $x \geq 5$

(4) $x < -9$

(2) $x > 2$

(5) $x \leq -2$

(3) $x \leq -6$

(6) $x \leq -10$

Quadratic Equations (degree of variable = 2)

A quadratic equation is an equation of the form: $ax^2 + bx + c = 0$ $a \neq 0$

To Solve a Quadratic Equation by Factoring

- (1) Set one side of the equation equal to zero
- (2) Factor the non-zero side of the equation
- (3) Set each factor equal to zero and solve

Examples:

(a) Solve: $x^2 + 4x + 3 = 0$

$$(x + 3)(x + 1) = 0$$

$$x + 3 = 0 \quad x + 1 = 0$$

$$x = -3 \quad x = -1$$

$$\begin{aligned}
 \text{(b) Solve: } 2x^2 - 7x &= -5 \\
 2x^2 - 7x + 5 &= 0 \\
 (2x - 5)(x - 1) &= 0 \\
 2x - 5 = 0 \quad x - 1 &= 0 \\
 2x = 5 \quad x &= 1 \\
 x &= \frac{5}{2}
 \end{aligned}$$

If a quadratic equation ($ax^2 + bx + c = 0$, $a \neq 0$) cannot be solved by factoring, we can use the quadratic formula:

a = the coefficient of the x^2 term
 b = the coefficient of the x term
 c = the constant term

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples:

$$\begin{aligned}
 \text{(a) Solve: } x^2 + 3x + 1 &= 0 \\
 a = 1, b = 3, c = 1 \\
 x &= \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2(1)} = \frac{-3 \pm \sqrt{9 - 4}}{2} = \frac{-3 \pm \sqrt{5}}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) Solve: } 2x^2 - 4x &= 3 \\
 2x^2 - 4x - 3 &= 0 \\
 a = 2, b = -4, c = -3 \\
 x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)} = \frac{4 \pm \sqrt{16 + 24}}{4} = \frac{4 \pm \sqrt{40}}{4}
 \end{aligned}$$

Simplify the radical

$$= \frac{4 \pm \sqrt{4 \times 10}}{4} = \frac{4 \pm 2\sqrt{10}}{4} = \frac{2(2 \pm \sqrt{10})}{4} = \frac{2 \pm \sqrt{10}}{2}$$

Problems

Solve

(1) $x^2 + 5x + 6 = 0$

(4) $x^2 + 5x - 1 = 0$

(2) $x^2 - 2x - 35 = 0$

(5) $2x^2 - x - 4 = 0$

(3) $3x^2 + 4x = 7$

(6) $x^2 + 6x = -2$

Answers

(1) $x = -3, x = -2$

(4) $x = \frac{-5 \pm \sqrt{29}}{2}$

(2) $x = -5, x = 7$

(5) $x = \frac{1 \pm \sqrt{33}}{4}$

(3) $x = -\frac{7}{3}, x = 1$

(6) $x = -3 \pm \sqrt{7}$